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9231/21

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

- 1 The curve C has polar equation $r = e^{\frac{3}{4}\theta}$ for $0 \leq \theta \leq \alpha$.

Given that the length of C is s , find α in terms of s .

[5]

[illegible]

- 2 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2 \sinh^2 x + 1. \quad [3]$$

[illegible]

- (b) Find the set of values of k for which $\cosh 2x = k \sinh x$ has two distinct real roots. [5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 The variables t and x are related by the differential equation

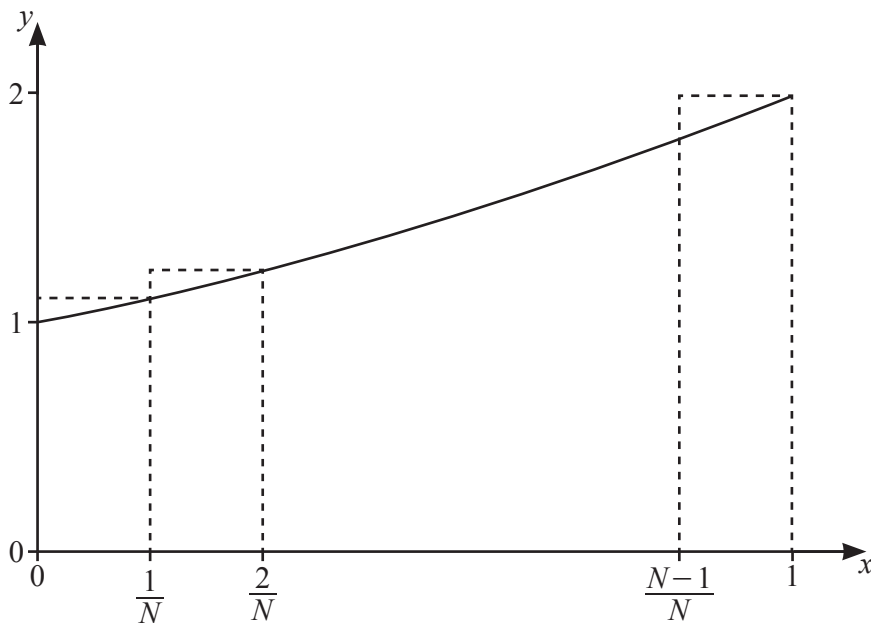
$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for x in terms of t . [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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- (b) Deduce an approximate value of $\frac{d^2x}{dt^2}$ for large positive values of t . [2]
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- 4 The diagram shows the curve with equation $y = 2^x$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.



- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 2^x dx < U_N$, where

$$U_N = \frac{2^{\frac{1}{N}}}{N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

[illegible]

- (b)** Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 2^x dx$. [4]

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- (c) Find the least value of N such that $U_N - L_N < 10^{-4}$. [2]

[illegible]

5 The variables x and y are such that $y = 0$ when $x = 0$ and

$$(x+1)y + (x+y+1)^3 = 1.$$

(a) Show that $\frac{dy}{dx} = -\frac{3}{4}$ when $x = 0$. [3]

[illegible]

(b) Find the Maclaurin's series for y up to and including the term in x^2 . [7]

[illegible]

- 6** Use the substitution $y = vx$ to find the solution of the differential equation

$$x \frac{dy}{dx} = y + \sqrt{9x^2 + y^2}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial in x .
[10]

[illegible]

7 (a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 7\theta = \frac{\operatorname{cosec}^7 \theta}{7 \operatorname{cosec}^6 \theta - 56 \operatorname{cosec}^4 \theta + 112 \operatorname{cosec}^2 \theta - 64}.$$

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\operatorname{cosec} q\pi$, where q is rational.

[5]

[illegible]

- 8 (a)** Find the value of a for which the system of equations

$$3x + ay = 0,$$

$$5x - y = 0,$$

$$x + 3y + 2z = 0,$$

does not have a unique solution.

[2]

[illegible]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

- (b)** Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$.

[7]

[illegible]

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- (c) Use the characteristic equation of $\mathbf{A}$ to show that

$$(\mathbf{A} + 6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

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